

# A Large Signal Analysis Leading to Intermodulation Distortion Prediction in Abrupt Junction Varactor Upconverters

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**Abstract**—Even though the abrupt junction varactor parametric upconverter is a “square-law” device, it exhibits gain saturation. This nonlinearity of the transfer characteristic is responsible for the nonlinear distortion present at the output of the device.

Of the several methods used to measure nonlinear distortion, the two tone test has been widely used. It is the accepted test method of both SMPTE and CCIR. This paper discusses the relationship between the nonlinear distortion measured by the two tone test and the nonlinear gain characteristic. It shows that if one is known the other is uniquely determined.

The analysis is broken into two parts. The first is a large signal analysis of the “square-law” parametric frequency converter. The results show that the gain is a function of the input signal level. It is this nonlinear relationship that is responsible for gain saturation. The second part is a study of the series representation of this nonlinear equation with respect to the terms containing the intermodulation frequencies. The final result is an equation which predicts the amplitude of the intermodulation distortion at any of the intermodulation frequencies. The analysis presented is closely related to the circuit and diode parameters and therefore not only predicts the amount of intermodulation distortion but also shows how it may be reduced. Experimental verification of the theory is also included.

## INTRODUCTION

THE FIDELITY of an amplifier or frequency converter is described by the linearity of its gain characteristic. This curve is almost a straight line over the usual dynamic range but deviates greatly from linearity after a certain input level is exceeded. This deviation from linearity is known as gain saturation. If an amplifier or converter is operated with signals in this region, the output becomes a distorted reproduction of the input. For high level applications, such as power sources or receiver front ends, located near transmitters, the specification of the upper limit of dynamic range, a specified percentage of gain saturation, for an amplifier or converter is as important as the specification of noise figure, the lower limit of dynamic range. Therefore, the design engineer must meet the dynamic range specifications set upon the equipment by the environment of the system.

An even more disturbing result of the nonlinearity of the device is the intermodulation distortion produced by

two inband signals. The nonlinear gain characteristic of an amplifier produces mixing of the two incoming signals and subsequently the output contains, in addition to the two original signals, responses at every frequency corresponding to  $(m+1)f_1 \pm mf_2$ , where  $f_1$  and  $f_2$  are the frequencies of the two original signals. That is, the output spectrum consists of equally spaced frequency responses, where the spacing is the difference between the two original frequencies. The output of a frequency converter would consist of the same spectrum except that the center is shifted by the frequency of the local oscillator or pump source.

Intermodulation distortion serves as a very convenient criteria for specifying the upper limit of dynamic range or the maximum allowable distortion of a device. To be more specific, the ratio of intermodulation distortion to the desired output, when two equal magnitude signals are applied to the device, is a measure of its linearity.

The nonlinear analysis presented in this paper is done with two primary purposes in mind. The first is to obtain an accurate gain equation for any type of signal, i.e., large signal or small signal, and the second is to analyze the gain equation so that the intermodulation distortion of the device can be found. The prediction of intermodulation distortion from the gain equation is perfectly general and any device which can be shown to have the same form of gain equation will have exactly the same intermodulation distortion properties.

An important aspect of the analysis presented in this paper is that the intermodulation distortion is related to the device and diode parameters. Therefore, the analysis not only predicts the amount of intermodulation distortion that is produced, but also provides a means of reducing it.

## DERIVATION OF THE GAIN EQUATION FOR A SQUARE-LAW PARAMETRIC UPCONVERTER

Gain saturation and intermodulation distortion are produced in square-law devices. The intermodulation distortion is due solely to the gain saturation and is unaffected by the higher order nonlinearity of the diode itself.

To show this, an abrupt junction diode ( $\gamma=1/2$ ) is

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assumed to be current-pumped. The specification of current pumping means that the current through the diode is sinusoidal. This is accomplished by using suitable filter networks. A current-pumped abrupt junction varactor has a linear variation of elastance with charge, which implies that the resulting terminal voltages are determined only by the second-order effect, i.e.,  $V \sim Q^2$ . The effect of the higher order terms, i.e.,  $V \sim Q^4$ , etc. do not appear and, therefore, have no effect on the following analysis [1], [2].

The circuit model of an upper sideband parametric frequency converter is shown in Fig. 1. The model is divided into two separate circuits. This is justified since any interaction between circuits takes place through the common elastance element.

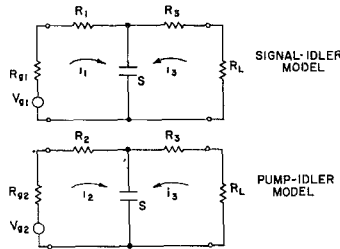


Fig. 1. Circuit model.

For simplicity, the loops of the circuit models are assumed to be resonant at their respective frequencies. The models are thus reduced to the source resistances  $R_{g1}$  and  $R_{g2}$ ; the circuit losses  $R_1$ ,  $R_2$ , and  $R_3$ ; and the time varying elastance  $S$ .

The diode current representations are

$$\begin{aligned} I_{\text{signal}} &= I_1 \cos(\omega_1 t) = i_1 e^{j\omega_1 t} + i_1 e^{-j\omega_1 t}, \\ I_{\text{pump}} &= I_2 \cos(\omega_2 t) = i_2 e^{j\omega_2 t} + i_2 e^{-j\omega_2 t}, \\ I_{\text{output}} &= I_3 \cos(\omega_3 t) = i_3 e^{j\omega_3 t} + i_3 e^{-j\omega_3 t}, \end{aligned}$$

where

$$\begin{aligned} i_k &= \frac{I_k}{2}, \\ k &= 1, 2, 3. \end{aligned}$$

The subscripts denote the various circuits the elements are associated with, i.e., 1 represents the signal, 2 the pump, and 3 the idler or output.

The signal output model leads to the following matrix relations between the signal and output:

$$\begin{bmatrix} V_{g1} \\ 0 \end{bmatrix} = \begin{bmatrix} R_{T1} & \frac{M_2 S_m}{\omega_3} \\ -\frac{M_2 S_m}{\omega_1} & R_{T3} \end{bmatrix} \begin{bmatrix} i_1 \\ i_3 \end{bmatrix}. \quad (1)$$

The pump output model leads to a similar matrix for the pump and output currents:

$$\begin{bmatrix} V_{g2} \\ 0 \end{bmatrix} = \begin{bmatrix} R_{T2} & \frac{S_m M_1}{\omega_3} \\ -\frac{S_m M_1}{\omega_2} & R_{T3} \end{bmatrix} \begin{bmatrix} i_2 \\ i_3 \end{bmatrix}. \quad (2)$$

$R_{T1}$ ,  $R_{T2}$ , and  $R_{T3}$  are the total resistances in the loops,

$$\begin{aligned} R_{T1} &= R_{g1} + R_1 \\ R_{T2} &= R_{g2} + R_2 \\ R_{T3} &= R_L + R_3. \end{aligned} \quad (3)$$

The frequency dependent terms represent the interactions between the two circuits. For example, the  $M_2 S_m / \omega_1$  term is derived from the capacitive reactance as follows. The reactance of the elastance at  $\omega_1$  is  $S / \omega_1$ . However,  $S$  can be shown to be equal to the product of the maximum elastance  $S_m$  and the ratio of instantaneous charge  $q$  to maximum charge  $Q_m$  [1]. This ratio  $q / Q_m$  has been defined [3] as a pumping quality factor  $M$ . With respect to the signal circuit, the elastance varies due to the charge variation in the pump circuit. Therefore, the reactance is

$$X_c = \frac{1}{\omega_1 C_2} = \frac{1}{\omega_1} S_2 = \frac{1}{\omega_1} S_m \frac{q_2}{Q_m} = \frac{1}{\omega_1} S_m M_2. \quad (4)$$

The other frequency terms can be derived in a similar manner.

Solving the first matrix for  $i_3 / i_1$  results in an expression for the current gain

$$G_i = \frac{i_3}{i_1} = \frac{1}{\omega_1} S_m M_2 \frac{1}{R_{T3}}. \quad (5)$$

Since  $M_2$  by definition is  $q_2 / Q_m$  and  $q_2 = i_2 / \omega_2$ , the gain equation can be written in terms of the current flowing in the pump circuit

$$G_i = \frac{S_m}{\omega_1 R_{T3}} \frac{i_2}{Q_m \omega_2} = K_0 i_2 \quad (6)$$

where

$$K_0 = \frac{S_m}{Q_m R_{T3} \omega_1 \omega_2}.$$

The value of  $i_2$  can be found from the second matrix.

$$\begin{aligned} i_2 &= \frac{V_{g2}}{Z_2} = \frac{V_{g2}}{R_{T2} + \frac{M_1^2 S_m^2}{\omega_2 \omega_3 R_{T3}}} \\ &= \frac{V_{g2}}{R_{T2}} \frac{1}{\left[ 1 + \frac{M_1^2 S_m^2}{\omega_2 \omega_3 R_{T2} R_{T3}} \right]}. \end{aligned} \quad (7)$$

Substituting this into the gain equation results in

$$G_i = K_0 \frac{V_{g2}}{R_{T2}} \frac{1}{\left[ 1 + \frac{M_1^2 S_m^2}{\omega_2 \omega_3 R_{T2} R_{T3}} \right]} \quad (8)$$

Now, since

$$M_1 = \frac{q_1}{Q_m} = \frac{i_1}{\omega_1} \frac{1}{Q_m}$$

$$G_i = K_0 \frac{V_{g2}}{R_{T2}} \frac{1}{\left[ 1 + \frac{S_m^2}{\omega_2 \omega_3 R_{T2} R_{T3}} \frac{i_1^2}{\omega_1^2 Q_m^2} \right]} \quad (9)$$

or

$$G_i = \frac{A_0}{1 + (A_1 i_1)^2} \quad (10)$$

where

$$A_0 = K_0 \frac{V_{g2}}{R_{T2}} = \frac{S_m}{Q_m R_{T3} \omega_1 \omega_2} \frac{V_{g2}}{R_{T2}}$$

$$A_1^2 = \frac{S_m^2}{\omega_1^2 Q_m^2} \frac{1}{\omega_2 \omega_3 R_{T2} R_{T3}}$$

It is important to note that both  $A_0$  and  $A_1$  are parameters and are constant for any specified signal and pump sources.

The small signal current gain can be derived from (10) by considering the case when  $(A_1 i_1)^2$  approaches zero or is negligible compared to unity. The small signal current gain is therefore  $A_0$ .

The magnitude of  $A_1$  determines how strongly the gain is a function of the input current. The smaller  $A_1$  is the larger  $i_1$  must become before  $(A_1 i_1)^2$  becomes appreciable compared to unity. It is interesting to note that the gain is never truly constant, even for very small values of  $(A_1 i_1)^2$ . Because of this, there is always intermodulation distortion present, even at extremely low input levels. However, in the region where the gain closely approximates the so-called small signal gain, the intermodulation distortion is never greater than 50 dB below the desired output level. This will be shown in succeeding sections.

The gain equation can also be expanded to form a series; therefore,

$$G_i = \frac{A_0}{1 + (A_1 i_1)^2}$$

$$= A_0 [1 - (A_1 i_1)^2 + (A_1 i_1)^4 - (A_1 i_1)^6 + \dots] \quad (11)$$

or

$$i_3 = A_0 [i_1 - (A_1 i_1)^2 i_1 + (A_1 i_1)^4 i_1 - (A_1 i_1)^6 i_1 + \dots] \quad (12)$$

or

$$i_3 = A_0 |i_1| [i_1 - |A_1 i_1|^2 i_1^3 + |A_1 i_1|^4 i_1^5 - |A_1 i_1|^6 i_1^7 + \dots] \quad (13)$$

where

$$\hat{i}_1 = \cos \omega_1 t.$$

#### INTERMODULATION ANALYSIS

The gain equation has been put in its series form, (13), for easy identification of the intermodulation terms. That is, when  $i_1$  is of the form  $\cos \omega_a t + \cos \omega_b t$ , the second term in the expansion gives rise to the desired output plus a contribution to the first intermod product term of the form  $K \cos (2\omega_a \pm \omega_b)t$ . In general, the intermod products of interest have frequencies

$$\omega_m = (m+1)\omega_a \pm m\omega_b \quad (14)$$

where  $m$  denotes the intermodulation product number.

Intermodulation products are usually referred to by the order of the particular nonlinear coefficient for which the intermodulation product appears. For example, the intermod due to the second term of the expansion would be called the third-order intermod product because that term is of the third order. However, since many terms of the expansion will be considered and since more than one term will contribute to intermodulation distortion at the same frequency, the intermod products will be referred to by the number corresponding to  $m$  in (14). Therefore,  $I_{P_m}$  where  $m=1$  is the first intermod product.

A convenient form of (13) would be one in which output current is expressed in terms of the ratio of output power to the maximum attainable output power  $P_0/P_{0m}$ . This is an easily measurable quantity which is also related to the conversion efficiency by a constant and is completely independent of signal match. To obtain the gain equation in terms of  $P_0/P_{0m}$ , the closed form of the gain equation is solved for the output current

$$i_3 = \frac{A_0 i_1}{1 + (A_1 i_1)^2} \quad (15)$$

The output power is

$$P_0 = 2R_L |i_3|^2 = 2R_L \left| \frac{A_0 i_1}{1 + (A_1 i_1)^2} \right|^2 \quad (16)$$

By obtaining the derivative of  $P_0$  with respect to  $i_1$ , the maximum value of  $P_0$  is found to be

$$P_{0m} = 2R_L \left( \frac{A_0}{2A_1} \right)^2 \quad (17)$$

The ratio  $P_0/P_{0m}$  is therefore,

$$\frac{P_0}{P_{0m}} = 4 \left| \frac{A_1 i_1}{1 + (A_1 i_1)^2} \right|^2. \quad (18)$$

Solving for  $|A_1 i_1|^2$ ,

$$|A_1 i_1|^2 = \frac{(1/2)(P_0/P_{0m})}{1 - (1/2)(P_0/P_{0m}) \pm \sqrt{1 - P_0/P_{0m}}}. \quad (19)$$

For simplicity this form of  $|A_1 i_1|^2$  will be denoted by  $y$ . That is, let

$$y = |A_1 i_1|^2 = \frac{(1/2)(P_0/P_{0m})}{1 - (1/2)(P_0/P_{0m}) \pm \sqrt{1 - P_0/P_{0m}}}. \quad (20)$$

Equation (13) can now be rewritten in terms of  $y$ ;

$$i_3 = A_0 |i_1| (i_1 - y i_1^3 + y^2 i_1^5 - y^3 i_1^7 + \dots). \quad (21)$$

The effect on the output current of two equal tones at the input can now be investigated. Let  $i_1$  consist of two equal tones of magnitude  $i$  and frequencies  $\omega_a$  and  $\omega_b$ , respectively.

$$i_1 = |i| (\cos \omega_a t + \cos \omega_b t). \quad (22)$$

Substituting this into (21) will result in a series of terms of the form

$$A_0 y^n (i_1)^{2n+1} \quad n = 0, 1, 2, 3, \dots \quad (23)$$

or

$$A_0 y^n [|i|^{2n+1} (\cos \omega_a t + \cos \omega_b t)^{2n+1}]. \quad (24)$$

The expansion of  $(i_1)^{2n+1}$  can become formidable especially when  $n$  takes on large values. It can be simplified by first evaluating the coefficients of  $(A+B)^{2n+1}$  in a binomial expansion, where

$$A = \cos \omega_a t; \quad B = \cos \omega_b t. \quad (25)$$

For values of  $n \neq 0$ , the binomial series will yield symmetrical products of the form

$$A^{2n+1} + B^{2n+1} + \sum_{k=m}^n b_k A^{2k} B^{2n+1-2k} + \sum_{k=m}^n b_k B^{2k} A^{2n+1-2k}. \quad (26)$$

Since the intermod products of interest have frequencies  $\omega_m = (m+1)\omega_a - m\omega_b$ , the term of interest in (26) is

$$\sum_{k=m}^n b_k A^{2k} B^{2n+1-2k} \quad (m = 1, 2, 3, \dots, n). \quad (27)$$

In terms of the trigonometric functions,

$$\sum_{k=m}^n [b_k (\cos^{2k} \omega_a t + \cos^{2n+1-2k} \omega_b t)]. \quad (28)$$

The only remaining calculation is obtaining the coefficients of the expansion for  $\cos^{2k} \omega_a t$  and  $\cos^{2n+1-2k} \omega_b t$ , which is a relatively simple matter;

$$\begin{aligned} \cos^{2k} \omega_a t &= \left(\frac{1}{2}\right)^{2k-1} \sum_{p=0}^P \frac{2k!}{(k-p)!p!} \cos 2(k-p)\omega_a t \\ &+ \left(\frac{1}{2}\right)^{2k} \frac{k!}{[(k/z)!]^2} \end{aligned} \quad (29)$$

$$\begin{aligned} \cos^{2n+1-2k} \omega_b t &= \left(\frac{1}{2}\right)^{2(n-k)} \sum_{p=0}^P \frac{(2n+1-2k)!}{(2n+1-2k-p)!p!} \\ &\cdot \cos (2n+1-2k-p)\omega_b t. \end{aligned} \quad (30)$$

Each term of interest in the series now becomes

$$A_0 y^n \left[ |i|^{2n+1} \sum_{k=m}^n b_k (\cos^{2k} \omega_a t + \cos^{2n+1-2k} \omega_b t) \right]. \quad (31)$$

Associated with each frequency yielded in the trigonometric expansion is its complement; i.e., for  $(m+1)\omega_a - m\omega_b$ , there is also a frequency  $(m+1)\omega_a + m\omega_b$ . However, the amplitudes of these terms are equal to the ones of interest and they represent symmetrical intermod sidebands.

The results of this procedure are tabulated in Table I. Calculations were performed for  $(\cos \omega_a t + \cos \omega_b t)$  to the seventeenth power. The values shown are for the intermod products before they are multiplied by their respective  $A_0 y^n$  coefficients.

The value of the coefficient  $y^n$  is dependent on the level of the input signal or as shown in (20) on the level of the output signal. The value of  $y$  has been calculated for several values of  $P_0/P_{0m}$  and is given in Table II. Since the only intermod frequencies that were considered were  $(m+1)\omega_a - m\omega_b$ , the output due to the signal at  $\omega_a$  is considered the desired output. That is, only half the spectrum has to be considered since the spectrum is symmetrical. However, the total input power for both tones has to be made equal to that of a single tone. Therefore for just one tone of a two-tone signal,  $y_a \rightarrow \frac{1}{2}y$ .

To obtain the net contribution of all the terms considered to the intermod products, the value of  $y$  from Table II is raised to the appropriate exponent and multiplied by the correct coefficient from Table I. All the  $I_{P_1}$  terms are then added or subtracted, depending upon the signs of their coefficients.

The intermod ratio is defined as the ratio of intermodulation distortion to desired output signal. Using the values obtained in the procedure described above, the intermod ratios were calculated and are shown in Table III for various values of  $P_0/P_{0m}$ .

TABLE I  
CONTRIBUTION TO INTERMOD OF TERMS IN THE EXPANSION DIVIDED BY  $A_0 y^n$

Intermod Product	Intermod Freq.	$i_1^3$	$i_1^5$	$i_1^7$	$i_1^9$	$i_1^{11}$	$i_1^{13}$	$i_1^{15}$	$i_1^{17}$
$S_0$	$\omega_a$	-2.25	6.25	-19.14	62.0	-208.4	718.9	-2527	9018
$IP_1$	$2\omega_a - \omega_b$	- .75	3.13	-11.48	41.3	-148.9	539.2	-1966	7214
$IP_2$	$3\omega_a - 2\omega_b$	0	0.63	- 3.83	17.7	- 74.4	299.5	-1179	4591
$IP_3$	$4\omega_a - 3\omega_b$	0	0	- 0.55	4.4	- 24.8	119.8	- 536	2295

$S_0$  represents the fundamental output at  $\omega_a$  or  $\omega_b$ .

TABLE II  
 $P_0/P_{0m}$  vs.  $y$

$P_0/P_{0m}$	$y$
.1	0.0132
.2	0.0281
.3	0.0448
.4	0.0635
.5	0.0862
.6	0.112
.7	0.143
.8	0.185
.9	0.258
1.0	0.500

TABLE III  
INTERMOD RATIO VALUES

$P_0/P_{0m}$	$S_0/A_0$	$I_{p1}/S_0$	$I_{p2}/S_0$	$I_{p3}/S_0$
0.1	-0.252 dB	-40.3 dB	-80 dB	-118.75 dB
0.2	-0.526 dB	-33.9 dB	-67.5 dB	-99.7 dB
0.3	-0.82 dB	-30.2 dB	-59.5 dB	-89.55 dB
0.4	-1.13 dB	-27.5 dB	-54.0 dB	-82.91 dB
0.5	-1.49 dB	-25.1 dB	-49.25 dB	-72.95 dB
0.6	-1.86 dB	-23.0 dB	-45 dB	-65.05 dB
0.7	-2.3 dB	-21.2 dB	-42.14 dB	-57.7 dB
0.8	-2.8 dB	-19.6 dB	-32.25 dB	

### THEORETICAL RESULTS

The data in Table III was used to plot the various intermod product ratios as functions of  $P_0/P_{0m}$ , Fig. 2. It should be noted that on these logarithmic plots, the intermod product ratios as functions of  $P_0/P_{0m}$  are straight lines. The slope of these lines can be expressed as

$$\text{Slope} = m[6 \text{ dB/octave}] \quad (32)$$

where  $m$  is the intermod product number. That is

- 1) For first product intermod ratio  $m=1$  and slope = 6 dB/octave.
- 2) For second product intermod ratio  $m=2$  and slope = 12 dB/octave.
- 3) For third product intermod ratio  $m=3$  and slope = 18 dB/octave.

In addition, for any specified value of  $P_0/P_{0m}$ , the second, third, fourth, etc., intermod product ratios are known if the first intermod product ratio is known.

$$\begin{aligned} & \left[ \begin{array}{l} m\text{th product intermod ratio in deci-} \\ \text{bels for any specified value of } P_0/P_{0m} \end{array} \right] \\ &= \left[ \begin{array}{l} \text{the product of } m \text{ and the first product intermod} \\ \text{ratio in decibels for the same value of } P_0/P_{0m} \end{array} \right]. \end{aligned}$$

For example, at  $P_0/P_{0m}=0.3$  the first product intermod ratio is -30 dB. Therefore,

$$\text{second product intermod ratio} = 2(-30) = -60 \text{ dB}$$

$$\text{third product intermod ratio} = 3(-30) = -90 \text{ dB}.$$

The nonlinearity of the gain equation indicates that the gain is reduced as the input becomes larger. The decrease in gain as the output becomes larger is shown quantitatively in Fig. 3, is a plot of large signal to small signal gain in decibels as a function of  $P_0/P_{0m}$ . The relationship between decreasing gain, or the nonlinearity of the gain curve, and intermodulation distortion has been determined by correlating the data of Figs. 2 and 3. The result is Fig. 4, which shows the first product intermod ratio as a function of gain nonlinearity.

Since conversion efficiency is an important parameter in frequency converters, it is convenient to express  $P_0/P_{0m}$  in terms of the conversion efficiency which is defined as

$$\eta = P_0/P_p \quad (33)$$

where  $P_p$  is the pump power. When  $P_0/P_{0m}$ ,

$$\eta_{\max} = \frac{P_{0m}}{P_p} \quad (34)$$

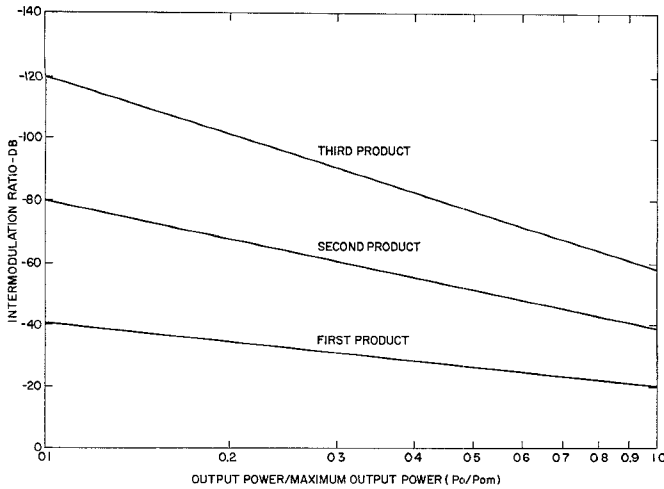


Fig. 2. Intermodulation distortion ratios.

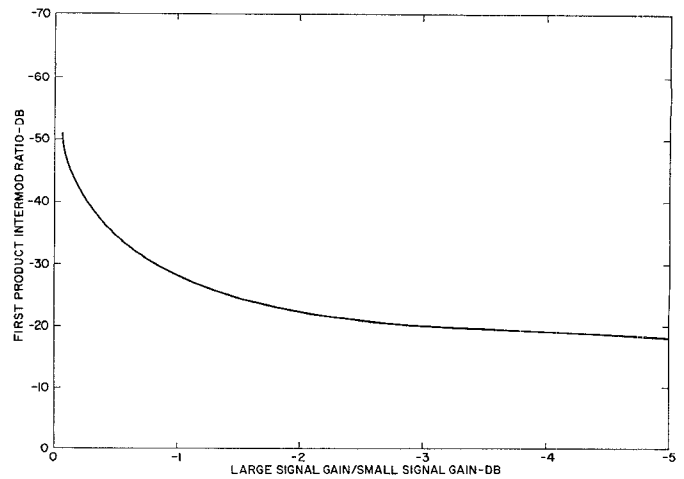


Fig. 4. First product intermodulation ratio as a function of gain nonlinearity.

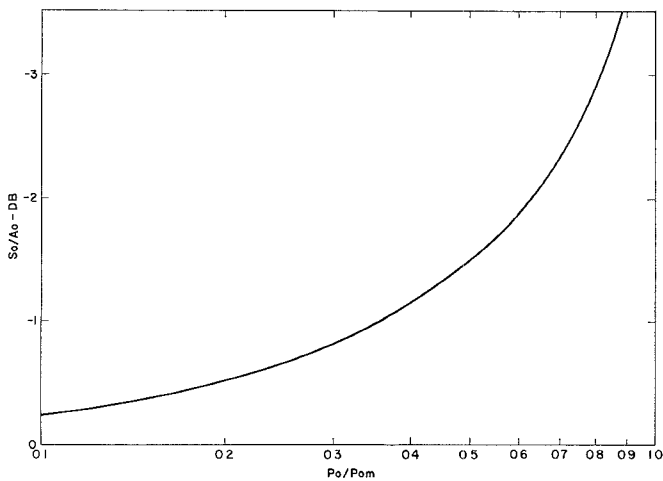


Fig. 3. Reduction of output power due to gain saturation.

or

$$\frac{\eta}{\eta_{\max}} = \frac{P_0}{P_{0m}} \quad (35)$$

Therefore, the  $P_0/P_{0m}$  axis for all the preceding curves can be relabeled the  $\eta/\eta_{\max}$  axis.

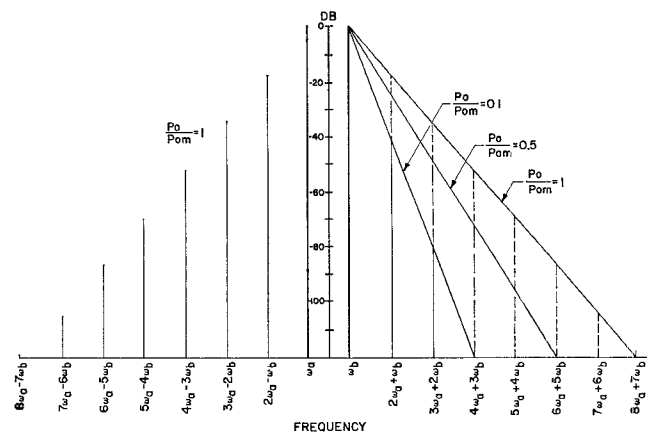
To get a better idea of the intermod frequencies being generated the spectral distributions (with respect to the pump frequency) of the output signal have been drawn for three values of  $P_0/P_{0m}$  in Fig. 5.

These representations of spectral response could only be obtained on a high-sensitivity zero-distortion spectrum analyzer. An actual analyzer would introduce additional distortion by its own mixing process.

#### EXPERIMENTAL RESULTS

Intermodulation measurements were made on a balanced C-band upconverter. The test setup is shown in Fig. 6. The varactor diodes used were Sylvania type D4251E. These diodes were tested and found to exhibit a nonlinearity coefficient of 0.42 over most of the voltage swing.

The pump source was a Klystron which delivered 1.4

Fig. 5. Relative amplitudes of spectral distribution for three values of  $P_0/P_{0m}$ .

watts at a frequency of 4230 Mc/s. The input signal channel was driven by two sources whose frequencies differed by 1 Mc/s at a center frequency of 310 Mc/s. Using the experimental data, the transfer characteristic of the device was drawn (Fig. 7). The maximum output power from this curve is shown to approach 250 mW. Using this value for  $P_{0m}$ ,  $P_0/P_{0m}$  was calculated for each point. The plot of first product intermod ratio as a function of  $P_0/P_{0m}$  for both the experimental and theoretical values is given in Fig. 8.

As the level of  $P_0$  is increased the experimental first product intermod ratio begins to increase in slope relative to the theoretical results. The increased distortion may be attributed to the products generated by the diode nonlinearity ( $\gamma \neq 0.5$ ). The decrease in small signal gain associated with large signal operation was also plotted using the experimental data. Since it is difficult to determine the small signal gain from the initial slope of the gain curve, it was assumed that the gain at  $P_0/P_{0m} = 0.1$  was down 0.25 dB from the small signal gain (this corresponds to the theoretical curve). The small signal gain was then found to be 12.9. Fig. 9 shows the experimental and theoretical values.

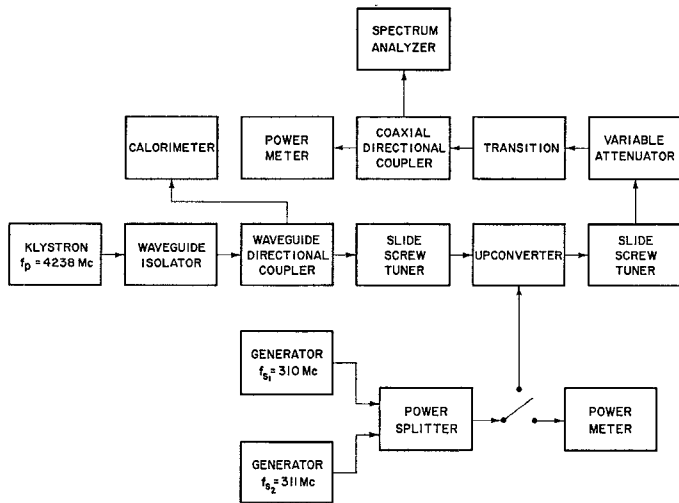


Fig. 6. Intermodulation test setup.

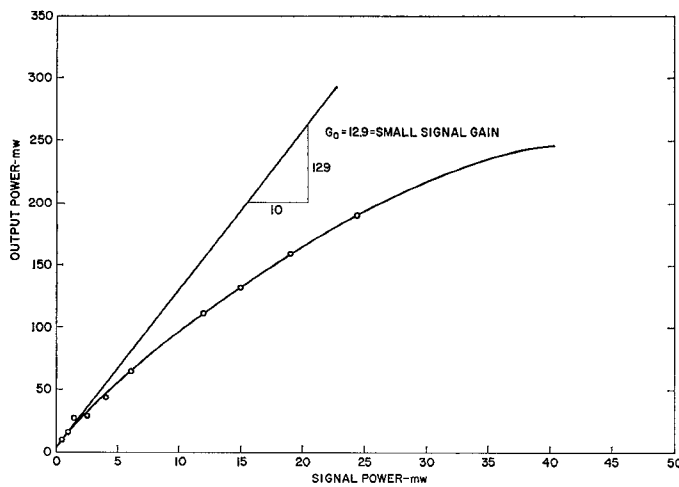


Fig. 7. Experimental gain characteristic.

### CONCLUSIONS

The analysis presented herein is based on a current-pumped abrupt junction (square-law) varactor diode. For this reason, the intermodulation distortion produced in frequency converters employing such diodes is due only to nonlinear gain saturation.

The assumption that all circuits were reactively tuned gives results for a broad-band circuit. If the device is made narrow band, by using high  $Q$  circuits, the intermodulation distortion products are reduced by the amount corresponding to the reduction in gain of the band-pass characteristic.

The equation for the first intermodulation distortion product ratio can be written down from the graph in Fig. 1.

$$IMR_1(\text{dB}) = \frac{I_{P_1}}{P_0} (\text{dB}) = 2 \frac{P_0}{P_{0m}} (\text{dB}) - 19.5. \quad (36)$$

In general, for the  $m$ th product,

$$IMR_m(\text{dB}) = \frac{I_{P_m}}{P_0} (\text{dB}) = m \left[ 2 \frac{P_0}{P_{0m}} (\text{dB}) - 19.5 \right]. \quad (37)$$

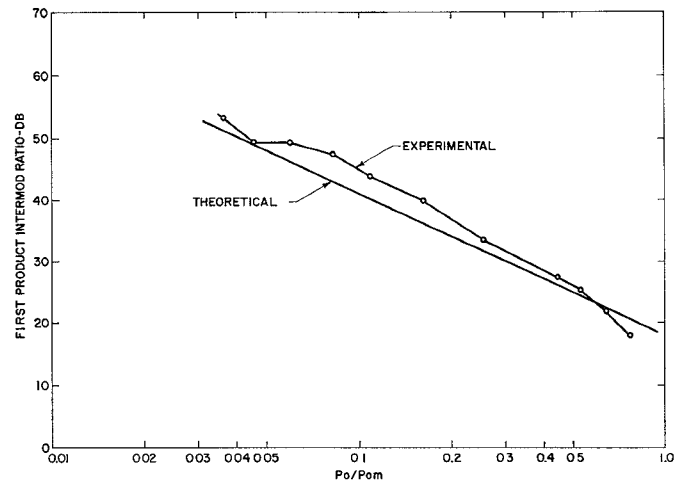


Fig. 8. Experimental first product intermodulation.

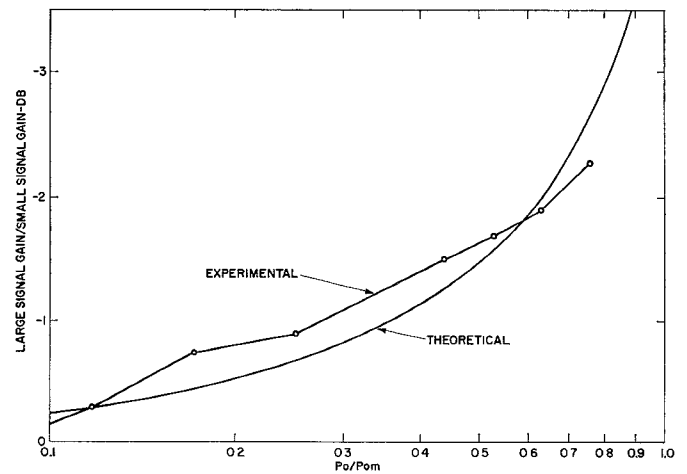


Fig. 9. Experimental values for ratio of large signal to small signal gain.

The intermodulation distortion of the upconverter is seen to be very closely related to  $P_0/P_{0m}$ . For example, if two devices have different small signal gains but the same  $P_{0m}$ , the one with the smaller gain will be able to handle a larger signal than the one with larger gain if the intermodulation distortion ratio is to be the same. Looking at it in a different way for the same level input signal, the intermodulation distortion will be greater for the converter with the higher gain.

If the small signal gain of two upconverters are the same, then the converter with the largest  $P_{0m}$  will have the least intermodulation distortion for the same input signal level. It is, therefore, seen that the intermodulation distortion can be reduced if  $P_{0m}$  is increased while keeping the small signal gain constant. It was shown in (17) that

$$P_{0m} = 2R_L \left( \frac{A_0}{2A_1} \right). \quad (17)$$

If  $A_0$  is to be kept constant, the only way of increasing  $P_{0m}$  is by decreasing  $A_1$ . This may be accomplished [see (10)] by increasing the resistive loading in the pump and output circuits while increasing the pump power so

that the small signal gain is preserved. It may also be accomplished by choosing a diode with a large  $Q_m/S_m$  ratio. In terms of diode parameters this ratio may be expressed as  $2C_{min}^2 V_B$  [2]. Therefore, the diode should have a large breakdown voltage  $V_B$  and largest possible capacitance at  $V_B$ .

If the values for  $A_0$  and  $A_1$  (10) are substituted into (17), it becomes

$$P_{0m} = \frac{V_g^2}{2R_L} \frac{\omega_3}{\omega_2} = P_{P_{avail.}} \frac{\omega_3}{\omega_2} \quad (38)$$

It has been assumed that the sources have been matched to the diode impedances (i.e.,  $R_{T_3} = 2R_L$ ,  $R_{T_2} = 2R_{g_2}$ ). The maximum obtainable output power is therefore the available pump power multiplied by the ratio of output frequency to pump frequency. If the pump power is increased 10 dB,  $P_{0m}$  also increases 10 dB. By examining (36), it is seen that increasing  $P_{0m}$  10 dB decreases the first intermodulation distortion product by 20 dB. The intermodulation distortion of a parametric upconverter can therefore be reduced by increasing the pump power; for every one-decibel increase in pump power the first intermodulation distortion product is reduced two decibels.

Intermodulation distortion can be predicted simply

by measuring the transfer characteristic, i.e.,  $P_0$  vs.  $P_{in}$ , and the frequency response of the device under test and then using (36). To compensate for the frequency response, (36) becomes

$$IMR_1(\text{dB}) = 2 \frac{P_0}{P_{0m}} (\text{dB}) - 19.5 - B, \quad (39)$$

where  $B$  is the attenuation of the bandpass characteristic at the intermodulation frequencies.

Equation (37) should not be considered the result for this particular device only. The intermodulation analysis was done for the gain equation, and is applicable to any device which has a gain equation of the same form.

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## The Effect of Parasitic Elements on Reflection Type Tunnel Diode Amplifier Performance

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**Abstract**—The effect of the tunnel diode series inductance and stray capacitance on the gain and bandwidth of broadband reflection type amplifiers is considered. General stability criteria imposed by these reactances are given together with realizability conditions for ideal (flat gain), Butterworth and Chebyshev responses. The main effect of the parasitic elements is to restrict the range of gain and bandwidth which may be achieved for a given number of elements in the matching network. The minimum gain is restricted together with both the maximum and minimum bandwidths. Comprehensive sets of curves are given which enable a rapid design of either Butterworth or Chebyshev response to be accomplished, and a procedure is given for conversion of the low-pass prototype network to band-pass form in the presence of the parasitic reactances. The frequency transformation is used to obtain an upper limit on the center frequency of the band-pass amplifier imposed by the parasitics. The use of the design data is illustrated by numerical examples.

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#### I. INTRODUCTION

THE BROADBAND tunnel diode reflection type amplifier has now reached the stage where it has become a competitor for many applications in microwave systems. With the production of diodes having ever increasing cutoff frequencies operation at  $X$  band and above is presently possible. In such high frequency diodes the junction capacitance is usually rather small being typically less than 0.5 pF, with the result that the parasitic elements of the diode (i.e., the series inductance and package capacitance) have reactances at the operating frequency which are of the same order as the reactance of the junction capacitance.

The synthesis of tunnel diode amplifiers has hitherto mainly depended on representing the diode as the parallel combination of a frequency independent capacitance and negative resistance over the frequency band